Edexcel Further Maths A-level

Further Pure 2

Formula Sheet

Provided in formula book

Not provided in formula book

Groups

Lagrange's Theorem	If H is a subgroup of a finite group G then $\lvert H\rvert$ divides $\lvert G\rvert$.

Further Calculus

Reduction Formulae

Integration by parts	$\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx} dx$

Arc Lengths

Arc length s on curve with Cartesian equation $y = f(x)$	$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$
Arc length s on the curve with Cartesian equation $x = f(y)$	$s = \int \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$
Arc length s on the parametrically defined curve $x = x(t), y = y(t)$	$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$
Arc length s on the curve defined by the polar equation $r = f(\theta)$	$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

Area of a Surface of Revolution

Area of the surface of revolution, S, of the Cartesian curve $y = f(x)$ after being rotated 2π radians about the x-axis	$S = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx$
Area of the surface of revolution, S, of the Cartesian curve $x = f(y)$ after being rotated 2π radians about the y-axis	$S = 2\pi \int_{y_A}^{y_B} x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$
Area of the surface of revolution, S, of the parametrically defined curve $x = x(t), y = y(t)$ after rotation around the x-axis:	$S = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$
Area of the surface of revolution, S, of the parametrically defined curve $x = x(t), y = y(t)$ after rotation around the y-axis:	$S = 2\pi \int x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$
Area of the surface of revolution, S, of the curve defined by the polar equation $r = f(\theta)$ rotated about the initial line, $\theta = 0$	$S = 2\pi \int r\sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$
Area of the surface of revolution, S, of the curve defined by the polar equation $r = f(\theta)$ rotated about the line $\theta = \pm\frac{\pi}{2}$	$S = 2\pi \int r\cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\ d\theta$

Further Matrix Algebra

Eigenvalues and Eigenvectors

An eigenvector of a matrix A is non-zero column vector $\boldsymbol{x}$, satisfying the equation $\boldsymbol{Ax} = \lambda\boldsymbol{x}$, where λ is a scalar called the eigenvalue corresponding to the eigenvalue $\boldsymbol{x}$.
The eigenvalues of A satisfy the characteristic equation $\det(\boldsymbol{A} - \lambda\boldsymbol{I}) = 0$

Form of the diagonal matrix, $\boldsymbol{D}$, of matrix $\boldsymbol{A}$	$\boldsymbol{D} = \boldsymbol{P}^{-1}\boldsymbol{AP}$ where $\boldsymbol{P}$ consists of the eigenvectors of $\boldsymbol{A}$, $\boldsymbol{D}$ has the respective eigenvalues of A on the leading diagonal

Cayley-Hamilton theorem	Every square matrix $\boldsymbol{M}$ satisfies its characteristic equation

Complex Numbers

Loci

For $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Loci of points z such that $\lvert z - z_1 \rvert = r$	Circle with centre (x_1, y_1) and radius r
Loci of points z such that $\lvert z - z_1 \rvert = \lvert z - z_2 \rvert$	Perpendicular bisector of the segment of the line joining z_1 and z_2
Loci of points z such that $\lvert z - a \rvert = k\lvert z - b \rvert$, where $a, b \in \mathbb{C}$ and $k \in \mathbb{R}, k > 0, k \neq 1$	Circle (find the centre and radius by finding the Cartesian equation)
Locus of points z such that $\arg(z - z_1) = \theta$	Half line from, but not including z_1 that has an angle θ with the line from z_1 parallel to the real axis
Locus of points z such that $\arg\left(\frac{z-a}{z-b}\right) = \theta,\ \theta \in \mathbb{R}, > 0$	Arc of a circle with endpoints at the points representing $a, b \in \mathbb{C}$

Number Theory

Bezout's Identity	If $a, b \ \neq 0, a, b \in \mathbb{Z}$, then there exists $x, y \ \in \mathbb{Z}$ such that $\gcd(a, b) = ax + by$

Fermat's Little Theorem	For p prime and $a \in \mathbb{Z}$ then $a^p \equiv a(\text{mod}\ p)$. If a is not divisible by p then $a^{p-1} \equiv 1(\text{mod}\ p)$.

Number of subsets of a set S with n elements	2^n
Number of permutations of n items	$n!$
Number of permutations of a selection of r items from a set of n items	${}^nP_r = \frac{n!}{(n-r)!}$
Number of permutations of n items, with r identical	$\frac{n!}{r!}$
Number of permutations of n items with sets of $r_1, r_2, \dots, r_n$ identical	$\frac{n!}{r_1! \times r_2! \times \dots \times r_n!}$
Number of combinations of r items from an original set of n	${}^nC_r = \frac{n!}{(n-r)!\,r!}$

Further sequences and series

First Order Recurrence Relations

Solution of the recurrence relation $u_n = au_{n-1}$	$u_n = u_0 a^n$ or $u_n = u_1 a^{n-1}$
Solution to the recurrence relation $u_n = u_{n-1} + g(n)$	$u_n = u_0 + \sum_{r=1}^{n} g(r)$

Particular Solutions for Recurrence Relations of the Form $u_n = au_{n-1} + g(n)$

Form of $g(n)$	Particular solution
p with $a \neq 1$	λ
$pn + q$ with $a \neq 1$	$\lambda n + \mu$
kp^n with $p \neq a$	λp^n
ka^n	$\lambda n a^n$
p with $a = 1$	λn
$pn + q$ with $a = 1$	$\lambda n^2 + \mu n$

Second Order Recurrence Relations

Particular Solutions for Recurrence Relations of the Form $u_n = au_{n-1} + bu_{n-2} + g(n)$, with Auxiliary Roots α and β

Form of $g(n)$	Form of particular solution
p, and $\alpha, \beta \neq 1$	λ
$pn + q$ and $\alpha, \beta \neq 1$	$\lambda n + \mu$
kp^n and $p \neq \alpha, \beta$	λp^n
p and $\alpha = 1, \beta \neq 1$	λn
$pn + q$ and $\alpha = 1, \beta \neq q$	$\lambda n^2 + \mu n$
p and $\alpha = \beta = 1$	λn^2
$pn + q$ and $\alpha = \beta = 1$	$\lambda n^3 + \mu n^2$
ka^n and $\alpha \neq \beta$	$\lambda n \alpha^n$
ka^n and $\alpha = \beta$	$\lambda n^2 \alpha^n$